	St Elizabeth's Catholic Voluntary Academy Calculation Policy	September 2023

Introduction

This policy outlines the strategies used for calculations taught and learnt at St Elizabeth's Voluntary Catholic Academy.

Within key stage 1 pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources [for example, concrete objects and measuring tools]. By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers. By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12-multiplication table and show precision and fluency in their work.

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio. At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

CPA approach

To support children with their calculating, the CPA approach will be adopted in all year groups. C stands for concrete, meaning children have access to manipulatives to support their working out, such as place value counters or numicon. P stands for pictorial, so children start to use visual representations such as bar models. A is abstract where children can confidently calculate without the use of concrete or visuals to support.

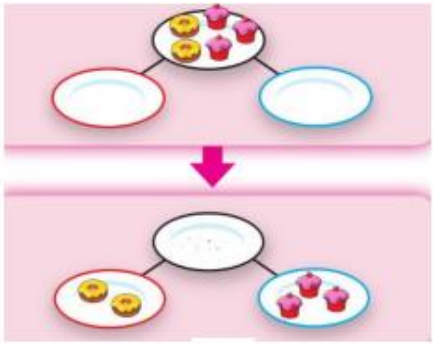
EYFS

Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. Children record their calculations in their own ways, there is no expectation of number sentences at this stage however children may choose this way to record their thinking.

Key language: whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus, less, more, group, share, equal, equals, is equal to, groups, equal groups, divide, share, shared equally

Addition: Children start to explore addition by sorting groups. They then use sorting to develop their understanding of parts and wholes. Children combine groups to find the whole, using a part-whole model to support their thinking. They also use the part-whole model to find number bonds within and to 10. Using a five frame and ten frame, children add by counting on. They start by finding one more before adding larger numbers using counters or cubes on the frames. Children use a number track to add by counting on. Linking this learning to playing board games is an effective way to support children's addition

Introducing the part-whole model Children sort everyday objects into parts.



One part is the

The other part is the

Subtraction: Children start to explore subtraction by sorting groups. They use sorting to develop their understanding of parts and wholes. When comparing groups, children use the language more than and fewer than. This will lead to finding the difference when they move into KS1. Children then connect subtraction with the idea of counting back and finding one less using a five frame to support their thinking. They explore subtraction by partitioning numbers, developing their understanding of parts and wholes. This links to their developing recall of number bonds. Children count back within 20 using number tracks and ten frames to see the effect of taking away.

Counting back and taking away (number track) Children use game boards and human number tracks to subtract by counting back.

9 take away 3 equals 6

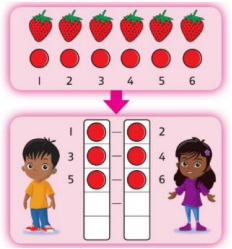
9…8…7…6

Multiplication and Division: Children first start to look at the idea of equal groups through their exploration of doubles. They use five frames and objects to check that groups are equal. Children then explore halving numbers by making 2 equal groups. They highlight patterns between doubling and halving seeing that double 2 is 4 and half of 4 is 2. As well as halving, children also explore sharing into more than 2 equal groups. They share objects 1 by 1, ensuring that each group has an equal share.

Making doubles Children explore doubles in their environment including in games such as on dominoes or dice. They focus on the understanding of doubles being 2 equal groups

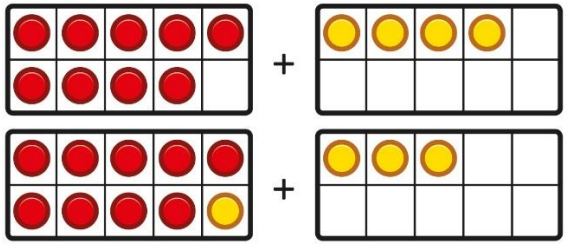
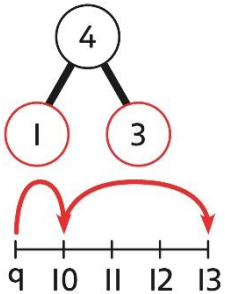
Double 4 is 8 Double 2 is 4 Double 3 is 6

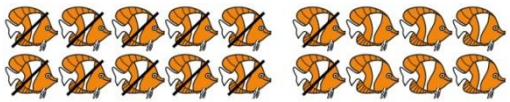
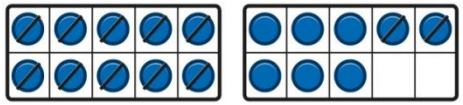
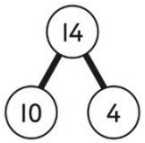
Halving and sharing Children use five frames to share amounts fairly and to check that the groups are equal. They share the counters/cubes one by one.



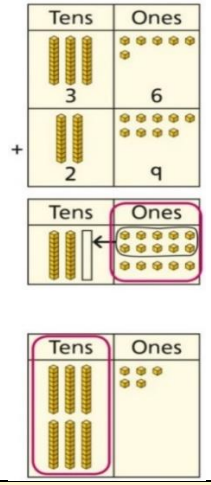
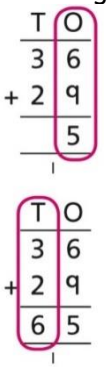
Half of 6 is 3

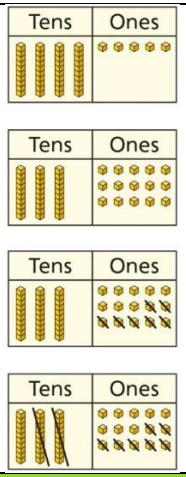
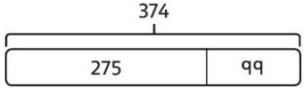
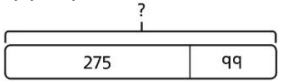
<table>
<tr><th colspan="4">Addition and Subtraction</th></tr>
<tr><td>Year 1</td><td colspan="3">Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations.

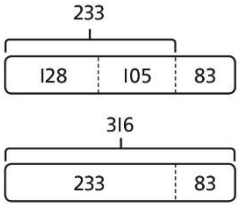
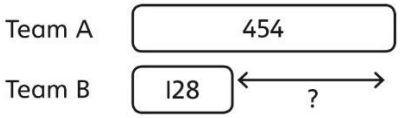
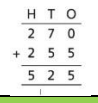
A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 – 3 and 15 – 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.</td></tr>
<tr><td>Addition</td><td>Concrete</td><td>Pictorial</td><td>Abstract</td></tr>
<tr><td>Bridging the 10 using number bonds</td><td>Children use a bead string to complete a 10 and understand how this relates to the addition.
 
7 add 3 makes 10.
So, 7 add 5 is 10 and 2 more</td><td>Children use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10.
 
.</td><td>Use a part-whole model and a number line to support the calculation.
 

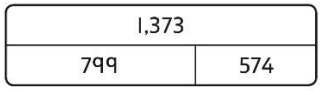
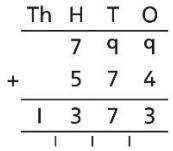
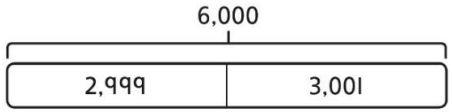
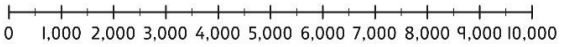
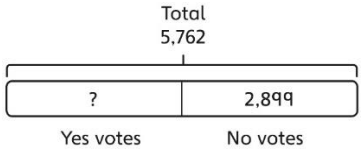
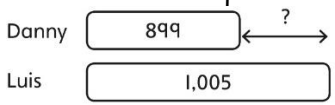
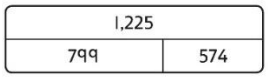
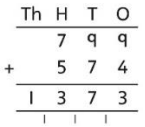
9 + 4 = 13</td></tr>
<tr><td>Subtraction</td><td>Concrete</td><td>Pictorial</td><td>Abstract</td></tr>
<tr><td>Subtracting 10s and 1s</td><td>For example: 18 – 12
Subtract 12 by first subtracting the 10, then the remaining 2.
 
First subtract the 10, then take away 2.</td><td>For example: 18 – 12
Use ten frames to represent the efficient method of subtracting 12.
 
First subtract the 10, then subtract 2.</td><td>Use a part-whole model to support the calculation.
 

19 – 14
19 – 10 = 9
9 – 4 = 5
So, 19 – 14 = 5</td></tr>
<tr><td colspan="4">Reasoning</td></tr>
</table>

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs	**Convince me** In my head I have two odd numbers with a difference of 2. What could they be? Convince me	**What else do you know?** If you know this: 12 – 9 = 3 what other facts do you know?	**Missing symbols** Write the missing symbols (+ - =) in these number sentences: 17 ☐ 3 ☐ 20 18 ☐ 20 ☐ 2
Year 2	**In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not wish to include it until Year 3.**		
Addition	**Concrete**	**Pictorial**	**Abstract**
Adding two 2-digit numbers with exchange	Add the 1s. Exchange 10 ones for a ten. Then add the 10s. 		2-digit numbers with exchange Add the 1s. Exchange 10 ones for a ten. Then add the 10s. 
Subtraction	**Concrete**	**Pictorial**	**Abstract**
Subtracting a 2-digit number with exchange		Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.	Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.

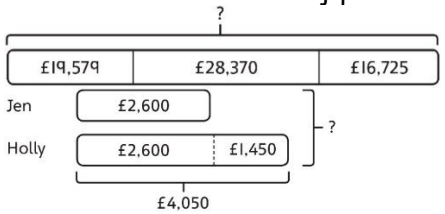
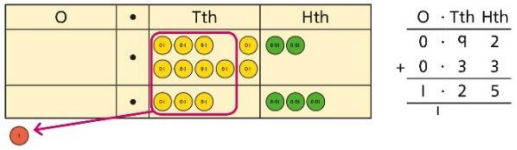
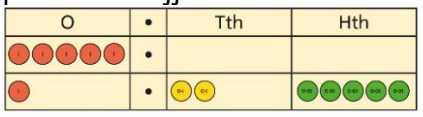
			T O 4 5 − 2 7 T O 3 15 − 2 7 T O 3 15 − 2 7 8 T O 3 15 − 2 7 1 8
Reasoning			
Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot	**Convince me** What digits could go in the boxes? 7 □ - □ = 46 Try to find all of the possible answers. How do you know you have got them all? Convince me	**What else do you know?** If you know this: 87 = 100 – 13 what other facts do you know?	**Missing symbols** Write the missing symbols (+ - =) in these number sentences: 80 □ 20 □ 100 100 □ 70 □ 30 87 □ 13 □ 100
Year 3	**In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply.**		
Addition	**Concrete**	**Pictorial**	**Abstract**
Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. *275 + 99 = ?*  *275 + 99 = 374*	Use representations to support choices of appropriate methods.  *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?*

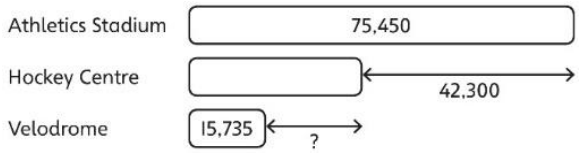
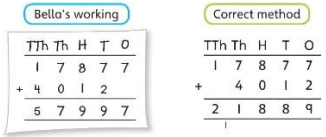
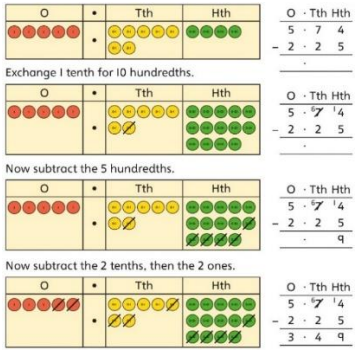
			I need to add three numbers. 128 + 105 = 233 
Subtraction	**Concrete**	**Pictorial**	**Abstract**
Representing subtraction problems		Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison.  Bar models can also be used to show that a part must be taken away from the whole.	Children use alternative representations to check calculations and choose efficient methods. Children use inverse operations to check additions and subtractions. The part-whole model supports understanding. *I have completed this subtraction.* *525 – 270 = 255* *I will check using addition.*  
Reasoning			
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction		**Convince me**  The total is 201 Each missing digit is either a 9 or a 1. Write in the missing digits. Is there only one way of doing this or lots of ways?	

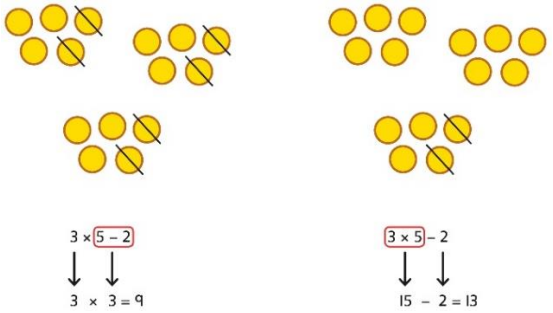
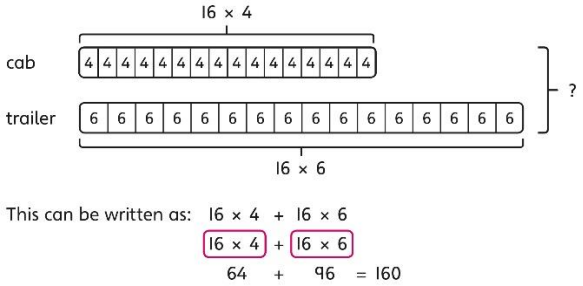
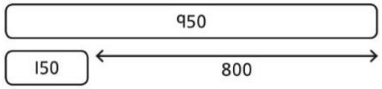
Year 4	In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.		
Addition	**Concrete**	**Pictorial**	**Abstract**
Representing additions and checking strategies		Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.   *I chose to work out 574 + 800, then subtract 1.*  *This is equivalent to 3,000 + 3,000.*	Use rounding and estimating on a number line to check the reasonableness of an addition.  *912 + 6,149 = ?* *I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.*
Subtraction	**Concrete**	**Pictorial**	**Abstract**
Representing subtractions and checking strategies		Use bar models to represent subtractions where a part needs to be calculated.  *I can work out the total number of Yes votes using 5,762 – 2,899.* Bar models can also represent 'find the difference' as a subtraction problem. 	Use inverse operations to check subtractions. *I calculated 1,225 – 799 = 574.* *I will check by adding the parts.*   *The parts do not add to make 1,225.* *I must have made a mistake.*
Reasoning			

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	**Convince me** ☐ - 666 = 8☐5 What is the largest possible number that will go in the rectangular box? What is the smallest?

Year 5	**Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage. Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.**

Addition	**Concrete**	**Pictorial**	**Abstract**
Representing additions		Bar models represent addition of two or more numbers in the context of problem solving.  Th H T O 2 6 0 0 + 1 4 5 0 4 0 5 0 Th H T O 2 6 0 0 + 4 0 5 0 6 6 5 0	Use approximation to check whether answers are reasonable. TTh Th H T O 2 3 4 0 5 + 7 8 9 2 2 0 2 9 7 TTh Th H T O 2 3 4 0 5 + 7 8 9 2 3 1 2 9 7 *I will use 23,000 + 8,000 to check.*
Adding decimals using column addition	Use place value equipment to represent additions. *Show 0·23 + 0·45 using place value counters.*	Use place value equipment on a place value grid to represent additions. Represent exchange where necessary.  O · Tth Hth 0 · 9 2 + 0 · 3 3 1 · 2 5 Include examples where the numbers of decimal places are different.  O · Tth Hth 5 · 0 0 + 1 · 2 5 6 · 2 5	Add using a column method, ensuring that children understand the link with place value. O · Tth Hth 0 · 2 3 + 0 · 4 5 0 · 6 8 Include exchange where required, alongside an understanding of place value. O · Tth Hth 0 · 9 2 + 0 · 3 3 1 · 2 5 Include additions where the numbers of decimal places are different. *3.4 + 0.65 = ?*

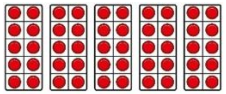
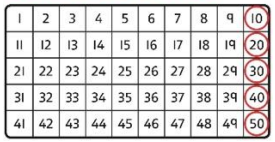
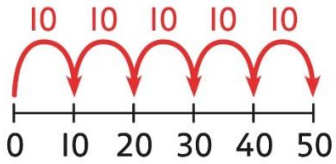
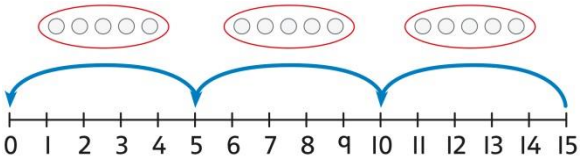
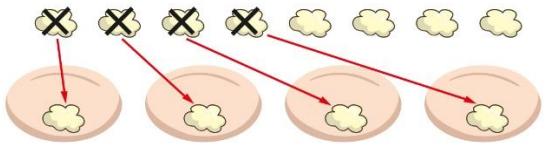
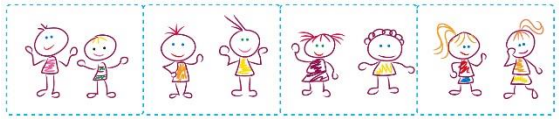
			O · Tth Hth 3 · 4 0 + 0 · 6 5 ·
Subtraction	**Concrete**	**Pictorial**	**Abstract**
Checking strategies and representing subtractions		Bar models represent subtractions in problem contexts, including 'find the difference'. 	Children can explain the mistake made when the columns have not been ordered correctly.  Use approximation to check calculations. *I calculated 18,000 + 4,000 mentally to check my subtraction.*
Subtracting decimals	Explore complements to a whole number by working in the context of length.  1 m – ☐ m = ☐ m *1 – 0·49 = ?*	Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 – 2·25 = ?* 	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *3·921 – 3·75 = ?* O · Tth Hth Thth 3 · 9 2 1 – 3 · 7 5 0 ·
Reasoning			
Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)		**Convince me** ☐ + 1475 = 6 ☐ 24 What numbers go in the boxes? What different answers are there?	

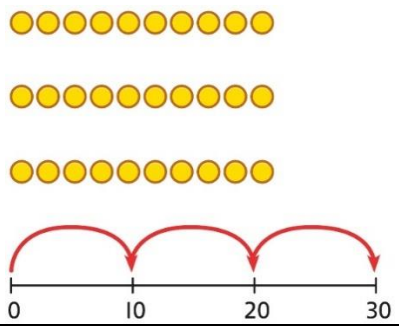
Year 6	**Children continue to compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.**		
Addition	**Concrete**	**Pictorial**	**Abstract**
Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. *3 × 5 − 2 = ?*  	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations.  	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. *4 + 6 × 16* *4 + 96 = 100* *(4 + 6) × 16* *10 × 16 = 160*
Subtraction	**Concrete**	**Pictorial**	**Abstract**
Subtracting mentally with larger numbers		Use a bar model to show how unitising can support mental calculations. *950,000 − 150,000* *That is 950 thousands − 150 thousands*  *So, the difference is 800 thousands.* *950,000 − 150,000 = 800,000*	Subtract efficiently from powers of 10. *10,000 − 500 = ?*
Reasoning			
Use their knowledge of the order of operations to carry out calculations involving the four operations	**Convince me** Three four-digit numbers total 12435. What could they be? Convince me	**What else do you know?** If you know this: 86.7 + 13.3 = 100 what other facts do you know?	**Missing symbols** Write the missing signs (+ - x ÷) in this number sentence: 6 ◯ 12.3 = 61.9 ◯ 11.9

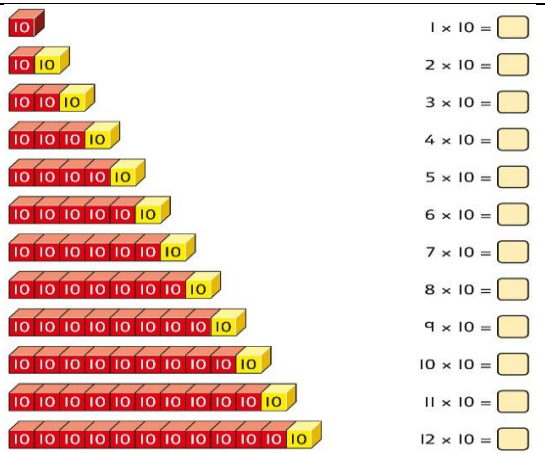
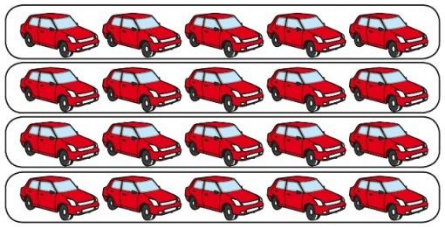
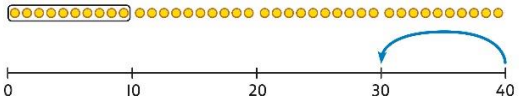
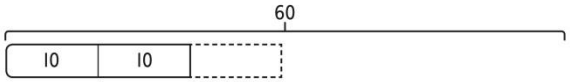
Multiplication and Division

Year 1

Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s.
In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole.

Multiplication	Concrete	Pictorial	Abstract
Finding the total of equal groups by counting in 2s, 5s and 10s	 There are 5 pens in each pack … 5…10…15…20…25…30…35…40…	100 squares and ten frames support counting in 2s, 5s and 10s.   1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50	Use a number line to support repeated addition through counting in 2s, 5s and 10s.  10 10 10 10 10 0 10 20 30 40 50
Division	**Concrete**	**Pictorial**	**Abstract**
Grouping	Learn to make equal groups from a whole and find how many equal groups of a certain size can be made. Sort a whole set people and objects into equal groups.  *There are 10 children altogether.* *There are 2 in each group.* *There are 5 groups.*	Represent a whole and work out how many equal groups.  *There are 10 in total.* *There are 5 in each group.* *There are 2 groups.*	Children may relate this to counting back in steps of 2, 5 or 10.  0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Sharing	Share a set of objects into equal parts and work out how many are in each part. 	Sketch or draw to represent sharing into equal parts. This may be related to fractions. 	10 shared into 2 equal groups gives 5 in each group.

Fractions	**Concrete**	**Pictorial**	**Abstract**
Reasoning			
Count in multiples of twos, fives and tens		**Practical** If we put two pencils in each pencil pot how many pencils will we need?	
Recognise find and name a half as one of two equal parts of an object, shape or quantity		**What do you notice?** Choose a number of counters. Place them onto 2 plates so that there is the same number on each half. When can you do this and when can't you? What do you notice?	
Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity		**True or false?** Sharing 8 apples between 4 children means each child has 1 apple.	
Year 2	**In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.** **In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator**		
Multiplication	**Concrete**	**Pictorial**	**Abstract**
Learning ×2, ×5 and ×10 table facts	Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.	Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.  	Understand how the times-tables increase and contain patterns.

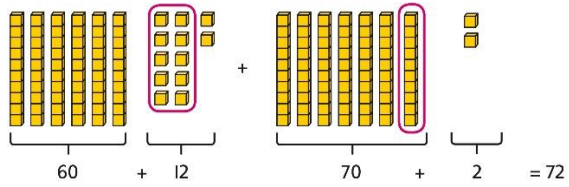
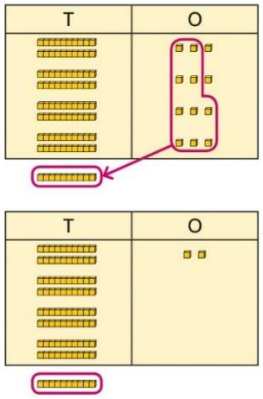
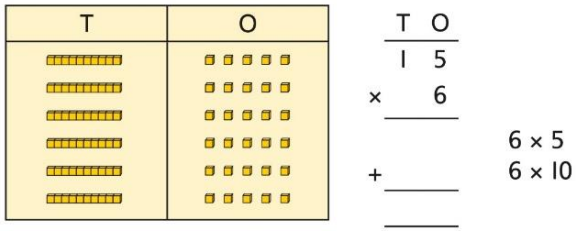
	 *3 groups of 10 … 10, 20, 30* *3 × 10 = 30*	*10 + 10 + 10 = 30* *3 × 10 = 30*	 *5 × 10 = 50* *6 × 10 = 60*
Division	**Concrete**	**Pictorial**	**Abstract**
Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.  *4 groups of 5 cars is 20 cars in total.* *20 divided by 4 is 5.*	Link equal grouping with repeated subtraction and known times-table facts to support division.  *40 divided by 4 is 10.* Use a bar model to support understanding of the link between times-table knowledge and division. 	Relate times-table knowledge directly to division. 1 × 10 = 10 2 × 10 = 20 **3 × 10 = 30** 4 × 10 = 40 5 × 10 = 50 6 × 10 = 60 7 × 10 = 70 8 × 10 = 80 I used the 10 times-table to help me. 3 × 10 = 30. *I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.* *3 × 10 = 30 so 30 ÷ 10 = 3*
Fractions	**Concrete**	**Pictorial**	**Abstract**
Reasoning			
Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs		**Prove It** Which four number sentences link these numbers? 3, 5, 15? Prove it.	

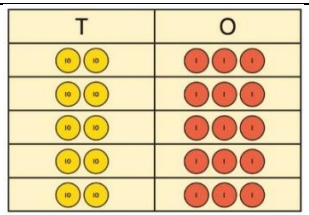
Recognise, find, name and write fractions $^{1}/_{3}$, $^{1}/_{4}$, $^{2}/_{4}$ and $^{3}/_{4}$ of a length, shape, set of objects or quantity	**What do you notice?** $\frac{1}{4}$ of 4 = 1 $\frac{1}{4}$ of 8 = 2 $\frac{1}{4}$ of 12 = 3 Continue the pattern What do you notice?	**True or false?** Half of 20cm = 5cm $\frac{3}{4}$ of 12cm = 9cm

Year 3

Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35.Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.

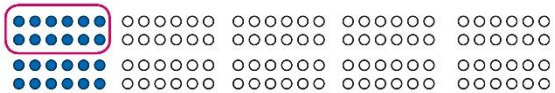
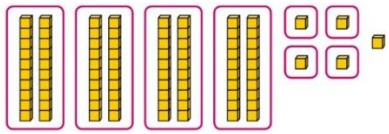
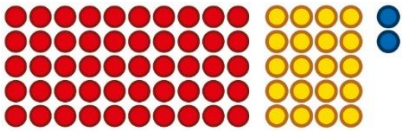
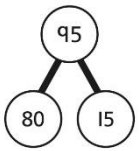
Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside.in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1.

Multiplication	Concrete	Pictorial	Abstract
Multiplying a 2-digit number by a 1-digit number, expanded column method	Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications. *3 × 24 = ?* *3 × 20 = 60* *3 × 4 = 12*  *3 × 24 = 60 + 12* *3 × 24 = 70 + 2*	Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s. *4 × 23 = ?*  *4 × 23 = 92*	Children may write calculations in expanded column form, but must understand the link with place value and exchange. Children are encouraged to write the expanded parts of the calculation separately.  *5 × 28 = ?*

	3 × 24 = 72	 *5 × 23 = ?* *5 × 3 = 15* *5 × 20 = 100* *5 × 23 = 115*	T O 2 8 × 5 4 0 5 × 8 1 0 0 5 × 20 1 4 0
Division	**Concrete**	**Pictorial**	**Abstract**
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder. *Make 29 from place value equipment.* *Share it into 2 equal groups.*  *There are two groups of 14 and 1 remainder.*	Use place value equipment to understand the concept of remainder in division. *29 ÷ 2 = ?*  *29 ÷ 2 = 14 remainder 1*	Partition to divide, understanding the remainder in context. *67 children try to make 5 equal lines.* *67 = 50 + 17* *50 ÷ 5 = 10* *17 ÷ 5 = 3 remainder 2* *67 ÷ 5 = 13 remainder 2* *There are 13 children in each line and 2 children left out.*

Reasoning

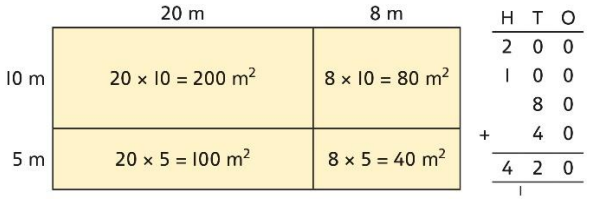
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers of times one-digit numbers, using mental and progressing to formal written methods	**Prove It** What goes in the missing box? x ? ? 4 80 12 Prove it. **How close can you get?**  Using the digits 2, 3 and 4 in the calculation above how close can you get to 100? What is the largest product? What is the smallest product?
Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one – digit numbers or quantities by 10.	**What do you notice?** 1/10 of 10 = 1 2/10 of 10 = 2 3/10 of 10 = 3

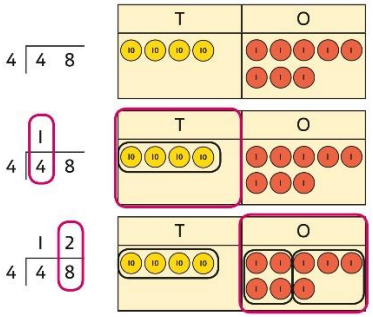
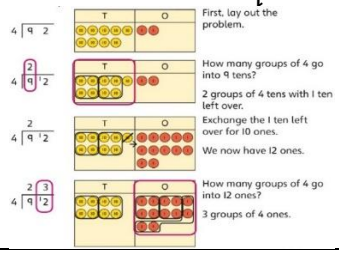
		Continue the pattern. What do you notice? What about 1/10 of 20? Use this to work out 2/10 of 20, etc.	
Year 4	**Continue and embed Year 3 for multiplication and division.** **In Year 4, children begin to work with fractions greater than 1. Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.**		
Multiplication	**Concrete**	**Pictorial**	**Abstract**
Multiplying more than two numbers	Represent situations by multiplying three numbers together.  *Each sheet has 2 × 5 stickers.* *There are 3 sheets.* *There are 5 × 2 × 3 stickers in total.* 5 × 2 × 3 = 30 10 × 3 = 30	Understand that commutativity can be used to multiply in different orders.  *2 × 6 × 10 = 120* *12 × 10 = 120* *10 × 6 × 2 = 120* 60 × 2 = 120	*Use knowledge of factors to simplify some multiplications.* *24 × 5 = 12 × 2 × 5* 12 × 2 × 5 = 12 × 10 = 120 So, 24 × 5 = 120
Division	**Concrete**	**Pictorial**	**Abstract**
Understanding remainders	Use place value equipment to find remainders. *85 shared into 4 equal groups* *There are 24, and 1 that cannot be shared.* 	Represent the remainder as the part that cannot be shared equally.  72 ÷ 5 = 14 remainder 2	Understand how partitioning can reveal remainders of divisions.  *80 ÷ 4 = 20* *12 ÷ 4 = 3* 95 ÷ 4 = 23 remainder 3

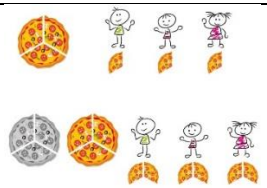
Reasoning	
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	**Prove It** What goes in the missing box? 6 ☐ x 4 = 512 Prove it. **How close can you get?** ☐☐☐ X 7 Using the digits 3, 4 and 6 in the calculation above how close can you get to 4500? What is the largest product? What is the smallest product?
Recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten	**What do you notice?** 1/10 of 100 = 10 1/100 of 100 = 1 2/10 of 100 = 20 2/100 of 100 = 2 How can you use this to work out 6/10 of 200? 6/100 of 200?

Year 5

Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value.

Multiplciation	Concrete	Pictorial	Abstract
Multiplying 2-digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. *23 × 15 = ?*  10 × 15 = 150 10 × 15 = 150  3 × 15 = 45 There are 345 bottles of milk in total. H T O 1 5 0 1 5 0 + 4 5 3 4 5 1	Use an area model and add the parts. *28 × 15 = ?*  H T O 2 0 0 1 0 0 8 0 + 4 0 4 2 0 1 *28 × 15 = 420*	Use column multiplication, ensuring understanding of place value at each stage. 3 4 × 2 7 2 3 8 (2) 34 × 7 3 4 × 2 7 2 3 8 (2) 34 × 7 6 8 0 34 × 20

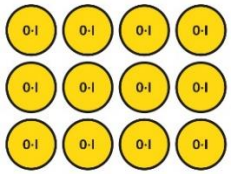
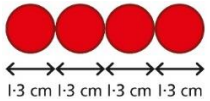
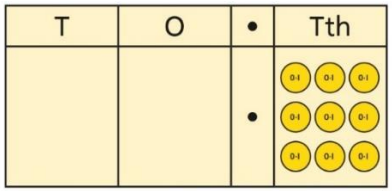
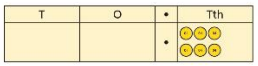
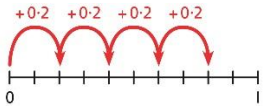
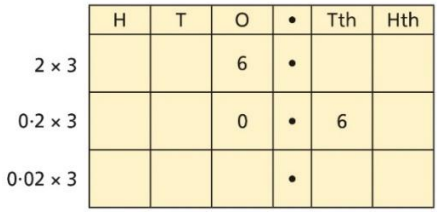
	23 × 15 = 345		3 4 × 2 7 2 3 8 (2) 34 × 7 6 8 0 34 × 20 9 1 8 34 × 27 1
Division	**Concrete**	**Pictorial**	**Abstract**
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. *268 ÷ 2 = ?* *There is 1 group of 2 hundreds.* *There are 3 groups of 2 tens.* *There are 4 groups of 2 ones.* *264 ÷ 2 = 134*	Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting.  Lay out the problem as a short division. *There is 1 group of 4 in 4 tens.* *There are 2 groups of 4 in 8 ones.* Work with divisions that require exchange.  	Use short division for up to 4-digit numbers divided by a single digit. 0 5 5 6 7 | 3 3 8 3 9 4 2 *3,892 ÷ 7 = 556* Use multiplication to check. *556 × 7 = ?* *6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892*
Understanding the relationship between	Use sharing to explore the link between fractions and division. *1 whole shared between 3 people.* *Each person receives one-third.*	Use a bar model and other fraction representations to show the link between fractions and division.	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$

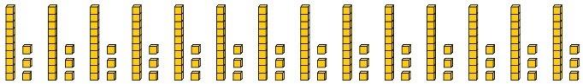
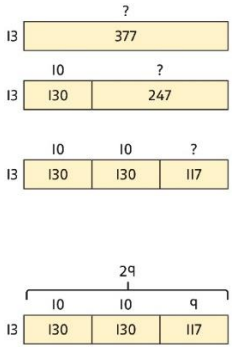
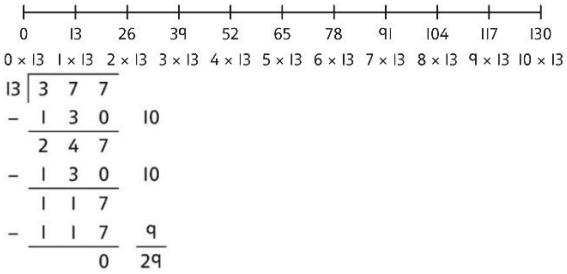
fractions and division		 $1 \div 3 = \frac{1}{3}$	$11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$

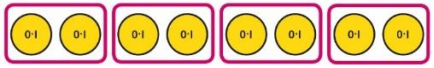
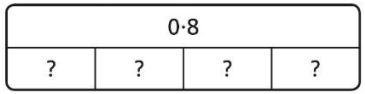
Reasoning		
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	**Prove It** What goes in the missing box? 12 ☐ 3 ÷ 6 = 212 12 ☐ 3 ÷ 7 = 212 22 ☐ 3 ÷ 7 = 321 r 6 323 x ☐ 1 = 13243	
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context		
Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents.	**What do you notice?** One tenth of £41 One hundredth of £41 One thousandth of £41 Continue the pattern What do you notice? 0.085 + 0.015 = 0.1 0.075 + 0.025 = 0.1 0.065 + 0.035 = 0.1 Continue the pattern for the next five number sentences.	**True or false?** 0.1 of a kilometre is 1m. 0.2 of 2 kilometres is 2m. 0.3 of 3 Kilometres is 3m 0.25 of 3m is 500cm. 2/5 of £2 is 20p

Year 6	
	In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6. : Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic. Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.

Addition	**Concrete**	**Pictorial**	**Abstract**
Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of measures.	Represent calculations on a place value grid.	Use known facts to multiply decimals. *4 × 3 = 12* *4 × 0·3 = 1·2*

	 *3 groups of 4 tenths is 12 tenths.* *4 groups of 3 tenths is 12 tenths.*  *4 × 1 cm = 4 cm* *4 × 0·3 cm = 1.2 cm* *4 × 1·3 = 4 + 1·2 = 5·2 cm*	3 × 3 = 9 3 × 0·3 = 0·9  Understand the link between multiplying decimals and repeated addition.  	*4 × 0·03 = 0·12* *20 × 5 = 100* *20 × 0·5 = 10* *20 × 0·05 = 1* Find families of facts from a known multiplication. *I know that 18 × 4 = 72.* *This can help me work out:* *1·8 × 4 = ?* *18 × 0·4 = ?* *180 × 0·4 = ?* *18 × 0·04 = ?* Use a place value grid to understand the effects of multiplying decimals. 

Subtraction	Concrete	Pictorial	Abstract
Dividing by a 2-digit number using long division	Use equipment to build numbers from groups.  *182 divided into groups of 13.* *There are 14 groups.*	Use an area model alongside written division to model the process. *377 ÷ 13 = ?* 	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. *377 ÷ 13 = ?* 

<table>
<tr>
<td></td>
<td></td>
<td>377 ÷ 13 = 29</td>
<td>A slightly different layout may be used, with the division completed above rather than at the side.
<pre>
   3
21 | 7 9 8
   - 6 3 0
     1 6 8
</pre>
<pre>
     3 8
21 | 7 9 8
   - 6 3 0
     1 6 8
   - 1 6 8
         0
</pre>
Divisions with a remainder explored in problem-solving contexts.</td>
</tr>
<tr>
<td>Dividing decimals</td>
<td>Use place value equipment to explore division of decimals.
 
8 tenths divided into 4 groups. 2 tenths in each group.</td>
<td>Use a bar model to represent divisions.
 
4 × 2 = 8 8 ÷ 4 = 2
So, 4 × 0·2 = 0·8 0·8 ÷ 4 = 0·2</td>
<td>Use short division to divide decimals with up to 2 decimal places.
<pre>
     .
8 | 4 · 2 4
</pre>
<pre>
    0 .
8 | 4 · 2 4
</pre>
<pre>
    0 · 5
8 | 4 · 2 2 4
</pre>
<pre>
    0 · 5 3
8 | 4 · 2 2 4
</pre></td>
</tr>
<tr>
<td colspan="4">Reasoning</td>
</tr>
<tr>
<td>Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.</td>
<td rowspan="2">Prove It
What goes in the missing box?
18 ☐ 4 ÷ 12 = 157
38 ☐ 5 ÷ 18 = 212.5
33 ☐ 2 ÷ 8 = 421.5
38 x ☐ .7 = 178.6</td>
<td rowspan="2">Can you find?
Can you find the smallest number that can be added to or subtracted from 87.6 to make it exactly divisible by 8/7/18?</td>
</tr>
<tr>
<td>Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context. Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</td>
</tr>
<tr>
<td>Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents.</td>
<td colspan="2">What do you notice?

One thousandth of my money is 31p. How much do I have?</td>
</tr>
</table>